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PHOTONEUTRINO REACTIONS IN A SUPERSTRONG MAGNETOACTIVE PLASMA

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ABSTRACT

The neutrino luminosity due to the photoneutrino process is computed in the presence of a superstrong magnetoactive electron plasma appropriate for neutron stars. The results indicate that for relatively low temperatures 10^8 K $\lesssim T < 5 \times 10^8$ K the energy loss rate is significantly reduced both in the low-density regime $\rho \lesssim 10^7$ g cm^{-3} (by the magnetic field) and in the high-density regime $\rho \gtrsim 10^7$ g cm^{-3} (by plasmon excitations). These effects are temperature dependent and they are less pronounced when 5×10^8 K $\leq T \leq 10^9$ K.

Subject headings: dense matter — hydromagnetics — neutrinos — nuclear reactions — plasmas — stars: neutron

I. INTRODUCTION

In recent years, there has been considerable interest in the effects brought about by intense magnetic fields of the order of 10^{12}–10^{13} gauss in astrophysical processes (Canuto *et al.* 1970; Canuto, Chiuderi, and Chou 1970; Canuto and Chou 1971; Chou 1971; Canuto and Fassio-Canuto 1973*a*, *b*; Canuto and Chou 1973; Canuto *et al.* 1974). The discovery of pulsars has greatly accelerated this trend. It is now generally believed that collapsed stellar objects such as dwarf stars and neutron stars are very likely to possess strong magnetic fields of the order of 10^8–10^{12} gauss. The significance of such superstrong fields is apparent in the models for pulsars proposed by most authors (Gold 1969; Gunn and Ostriker 1969; Chiu and Canuto 1971).

The origin of ultrastrong magnetic fields in astrophysics is usually attributed to flux conservation during gravitational collapse of magnetic stars (Woltjer 1964). Alternatively, intense magnetic fields may be maintained in the interiors of neutron stars either by Landau orbital ferromagnetism of the degenerate electrons (Lee *et al.* 1969) or by neutron ferromagnetism (Brownell and Callaway 1969; Silverstein 1969; Østgaard 1970). Magnetic fields of the order of 10^{12} gauss could be maintained in the interiors of neutron stars if Landau orbital ferromagnetism of the electrons operates, whereas interior fields of the order of 10^{15} gauss or greater may be generated if there is neutron ferromagnetism.

The effects of superstrong magnetic fields on atomic processes have been emphasized by Ruderman (1971) and Spruch, Mueller, and Rau (1971). Weak interactions in strong magnetic fields have been studied by several authors. The relevant neutrino reactions in strong magnetic fields that have already been investigated are summarized by Canuto *et al.* (1974). The effects of a strong magnetic field on the photoneutrino process were considered by Canuto and Fassio-Canuto (1973*b*). In particular they studied the special case in which a photon propagates along the magnetic field.

In this paper, we consider the photoneutrino reaction in a strongly magnetized plasma. The presence of an anisotropic magnetized plasma gives rise to interesting new features. We note that even in the absence of a strong magnetic field the inclusion of plasma effects in the photoneutrino reaction is qualitatively very different from that in which the plasma effects are neglected (Beaudet, Petrosian, and Salpeter 1967). Propagation of electromagnetic waves in a magnetoactive plasma is very sensitive to the direction of propagation relative to the external magnetic field. For instance, propagation along the magnetic field gives rise to circularly polarized transverse waves whereas propagation across the magnetic field results in linearly polarized mixed types of waves which are partially transverse and partially longitudinal. It is clear that the photoneutrino energy loss rates are qualitatively very different for incoming photons propagating parallel or perpendicular to the external magnetic field.

In § II of this paper we formulate the description of photoneutrino emission in a superstrong magnetoactive degenerate electron plasma in the language of quantum field theory, and consider in detail, at temperatures, densities, and magnetic field strengths expected for neutron stars, the photoneutrino luminosity due to the various modes of plasma excitations at an arbitrary angle relative to the static homogeneous magnetic field. In § III we specialize our general results for the neutrino luminosity to the important special cases in which the incident plasma waves are propagated along and across the external magnetic field. Finally, in § IV we discuss the possible application of our results to the cooling of pulsars and elaborate in detail their consequences and modifications introduced by the presence of the magnetized electron plasma.

II. PHOTONEUTRINO LUMINOSITY IN A SUPERSTRONG MAGNETOACTIVE PLASMA

The conserved vector current hypothesis for the $V - A$ theory of weak interactions proposed by Feynman and Gell-Mann (1958) predicts among other things the possible existence of the direct electron-neutrino pair interactions. Among the possibilities opened up by the proposal of the direct $(e\nu)$ $(e\nu)$ pair interaction is a process that has had primary importance for the thermal history of dense stellar objects like neutron stars; such a process is derived as a modification of the ordinary Compton scattering if, at the second vertex, the scattered photon is replaced by a neutrino-antineutrino pair. The effects introduced by the presence of a superstrong magnetic field for the photoneutrino process has been considered by Canuto and Fassio-Canuto (1973*b*), who neglected the effects of the presence of the background plasma.

In this paper we shall reformulate the neutrino-antineutrino pair energy loss rates in such a way that the effects arising from the presence of the superstrong magnetoactive plasma are explicitly taken into account.

We now compute a general expression for the photoneutrino luminosity as a function of temperature, density, and magnetic field strength for incoming plasmons propagated at an arbitrary angle relative to the external magnetic field. We choose Cartesian coordinates such that the static, homogeneous magnetic field is oriented along the z-axis whereas the propagation vector $\boldsymbol{k}_\gamma$ for the incident plasmon is confined to the x-z plane, namely, $\boldsymbol{k}_\gamma = (k_\gamma \sin\theta, 0, k_\gamma \cos\theta)$, where θ is the angle between the propagation vector $\boldsymbol{k}_\gamma$ and the external magnetic field.

Consider the process, for instance, in the interiors of dense stellar objects such as neutron stars in which an electron in the initial Landau state $|i\rangle = |n, s, p_z\rangle$ absorbs an incoming transverse plasmon described by the electromagnetic potential A_μ at the spacetime vertex X, and makes a transition to the final state $|f\rangle = |n', s', p_z'\rangle$ by emitting a neutrino-antineutrino pair at the second spacetime vertex Y. This process is diagrammatically represented in Figures 1*a* and 1*b*, where the double lines are the Landau states for the electrons, the wiggly line represents the plasmon excitations, and the dashed lines denote the neutrino pairs.

The Lagrangian describing the system is then given by

$$\mathscr{L}(x) = \mathscr{L}_W(x) + \mathscr{L}_\gamma(x)\,, \qquad \mathscr{L}_W(x) = 2^{-1/2} g[\bar{\psi}_\nu(x)\Gamma_\mu\psi_\nu(x)][\bar{\psi}_e(x)\Gamma_\mu\psi_e(x)]\,,$$
$$\mathscr{L}_\gamma(x) = -ie\bar{\psi}_e(x)\gamma_\lambda A_\lambda(x)\psi_e(x)\,, \tag{1}$$

where

$$\Gamma_\mu \equiv \gamma_\mu(1 + \gamma_5)\,, \qquad \bar{\psi} = \psi^+\gamma_4\,,$$

where the γ-matrices are taken in the standard Dirac-Pauli representation.

The part of the S-matrix describing the process in Figure 1*b* is

$$S = (\hbar c)^{-2} \int d^4x \int d^4y T\{\mathscr{L}_\gamma(x)\mathscr{L}_W(y)\}\,, \tag{2}$$

where T is the time-ordering operator. The S-matrix element corresponding to the direct diagram shown in Figure 1*a* is then given by

$$S_D = \frac{ieg}{2^{1/2}(\hbar c)^2} \int d^4x \int d^4y [\bar{\psi}_\nu(y)\Gamma_\mu\psi_\nu(y)]\bar{\psi}_f(y)\Gamma_\mu G(x, y)\gamma_\lambda A_\lambda(x)\psi_i(x)\,, \tag{3a}$$

whereas that for the exchange diagram in Figure 1*b* is described by

$$S_E = \frac{ieg}{2^{1/2}(\hbar c)^2} \int d^4x \int d^4y [\bar{\psi}_\nu(y)\Gamma_\mu\psi_\nu(y)]\bar{\psi}_f(x)\gamma_\lambda A_\lambda(x)G(y, x)\Gamma_\mu\psi_i(y)\,, \tag{3b}$$

where $G(x, y)$ is the propagator from the spacetime point x to the spacetime point y.

To proceed further, we shall at this point make the simplifying assumption of treating the electrons in the nonrelativistic approximation. This is due partly to the fact that the Green function for the relativistic electrons in an external magnetic field is rather complicated to use. Second, the normalization factor (to be described shortly) of the electromagnetic potential A_μ for the plasmon excitations in a magnetized electron gas depends on the dielectric tensor and the dispersion relations. Moreover, from the point of view of full-fledged many-body theory, relativistic quantum-mechanical dispersion relations for a magnetoactive plasma do not exist as yet.

In order to clearly define the region of validity of the present computation, let us note that for a particle in a magnetic field, the sum over the quantum number n (see eq. following [28]) can be replaced by an integration over n when the average value of x^2 is much greater than 2θ, $\theta = H/H_q$. This means that the criteria for a gas to be treated as magnetized are

$$p_z^2/m^2c^2 \leq 2H/H_q$$

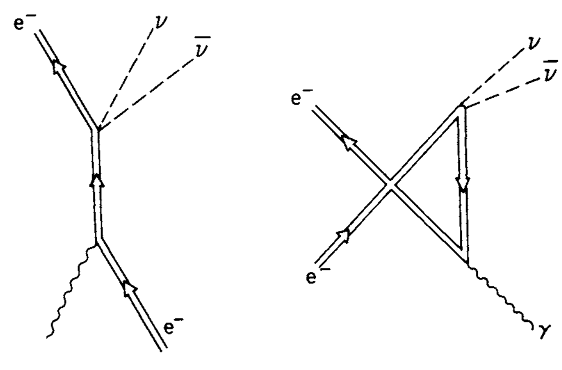

FIG. 1.—Feynman diagrams for the photoneutrino processes in a superstrong magnetoactive electron plasma. The double lines are the Landau states for the electrons, the wiggly line represents the plasmon excitations, and the dashed lines denote the neutrino pairs.

or

$$kT/mc^2 \lesssim 2H/H_q \quad \text{(nonrelativistic)}\,, \tag{4a}$$

$$(kT/mc^2)^2 \lesssim 2H/H_q \quad \text{(relativistic)}\,. \tag{4b}$$

Since for a degenerate gas, we have

$$3p_z^2 \sim 2m\epsilon_F \quad \text{(nonrelativistic)}\,, \tag{4c}$$

$$3p_z c \sim \epsilon_F \quad \text{(relativistic)}\,, \tag{4d}$$

the following criteria follow for magnetic effects to be relevant:

$$\frac{2}{3}\frac{\epsilon_F}{mc^2} \lesssim 2H/H_q \quad \text{(nonrelativistic)}\,,$$

$$\frac{1}{3}\left(\frac{\epsilon_F}{mc^2}\right)^2 \lesssim 2H/H_q \quad \text{(relativistic)}\,, \tag{5}$$

or

$$(\rho_7/\mu)^{2/3} \leq 2H/H_q\,. \tag{6}$$

Clearly (6) includes both the relativistic and the nonrelativistic cases.

The Green function for an electron in a strong magnetic field may be written as (Canuto and Fassio-Canuto 1973*b*)

$$G(x, y) = \frac{\hbar}{2\pi i}\int_{-\infty}^{\infty} d\omega \exp\,[i\omega(t_x - t_y)] \sum_I \frac{\psi_I(y)\psi_I^*(x)}{E_I - \hbar\omega + i\eta}\gamma_4\,, \tag{7}$$

where ψ_I represents the Landau wave-function for the electron in the intermediate state and the subscript I denotes all the quantum numbers to specify the electron state in the presence of an external magnetic field. More specifically, $|I\rangle = |n'', s'', p_z''\rangle$, where n'', s'', p_z'' denote the orbital quantum number, the spin projection, and momentum along the magnetic field, respectively.

Substituting equation (7) into equations (3a) and (3b) and performing the indicated integrations, we obtain

$$S_D = \frac{2\pi egN_\gamma^{1/2}}{2^{1/2}\Omega_\nu}\,\delta(E_i + E_\gamma - E_f - E_\nu - E_{\bar\nu})$$

$$\times \sum_I \frac{\langle f|\gamma_4\Gamma_\mu \exp\,[i(\boldsymbol{p} + \boldsymbol{q})\cdot\boldsymbol{y}/\hbar]|I\rangle\langle I|\gamma_4\gamma_\gamma e_\gamma \exp\,(i\boldsymbol{k}_\gamma\cdot\boldsymbol{x})|i\rangle}{E_I - E_i - E_\gamma + i\eta}\,\bar{U}_\nu(p)\Gamma_\mu U_\nu(q)\,, \tag{8}$$

$$S_E = \frac{2\pi egN_\gamma^{1/2}}{2^{1/2}\Omega_\nu}\,\delta(E_i + E_\gamma - E_f - E_\nu - E_{\bar\nu})$$

$$\times \sum_I \frac{\langle f|\gamma_4\gamma_\lambda e_\lambda \exp\,(i\boldsymbol{k}_\gamma\cdot\boldsymbol{x})|I\rangle\langle I|\gamma_4\Gamma_\mu \exp\,[i(\boldsymbol{p} + \boldsymbol{q})\cdot\boldsymbol{y}/\hbar]|i\rangle}{E_I - E_f + E_\gamma + i\eta}\,\bar{U}_\nu(p)\Gamma_\mu U_\nu(q)\,, \tag{9}$$

where $\boldsymbol{p}$ and $\boldsymbol{q}$ denote the momenta for the neutrino-antineutrino pair and E_i, E_f are the energy eigenvalues for the electrons in an external magnetic field corresponding to the initial and the final states, respectively.

The electromagnetic potential A_μ for the transverse plasmons may be written in the form (Canuto, Chiuderi, and Chou 1970)

$$A_\mu(x) = N_\gamma^{1/2} e_\mu \exp(ik_\gamma x)\,, \qquad N_\gamma \equiv \frac{2\pi(\hbar c)^2}{E_\gamma \Omega_\gamma} |\zeta_{ij}| \equiv \frac{2\,\mathrm{Tr}\,\lambda_{ij}}{E_\gamma \partial \|\Lambda_{ij}\| / \partial E_\gamma}\,, \qquad E_\gamma \equiv \hbar\omega_\gamma\,, \tag{10}$$

where λ_{ij} is the cofactor of the Maxwell operator Λ_{ij} defined by

$$\Lambda_{ij} = \frac{c^2 K^2}{\omega^2}(K_i K_j - \delta_{ij}) + \epsilon_{ij}\,, \qquad \boldsymbol{K} = \boldsymbol{k}_\gamma / |\boldsymbol{k}_\gamma|\,, \tag{11}$$

and ϵ_{ij} is the dielectric tensor for the plasma. We note that the plasma parameters are contained in the factor ζ_{ij} which becomes unity in the vacuum limit. In the absence of an external magnetic field it can also be shown that equation (10) reduces to the well known expressions given by Adams, Ruderman, and Woo (1963).

Let us now consider the nonrelativistic limit of the product $\psi_i^*(x)\gamma_4\gamma_\lambda e_\lambda \psi_i(x)$. A simple calculation shows that

$$\langle I|\gamma_4\gamma_\lambda e_\lambda \exp(i\boldsymbol{k}_\gamma\cdot\boldsymbol{x})|i\rangle \approx \left\langle I\left|\frac{1}{mc}\,\boldsymbol{\Pi}\cdot\boldsymbol{e}\exp(i\boldsymbol{k}_\gamma\cdot\boldsymbol{x})\right|i\right\rangle. \tag{12}$$

This is because the quantity $\gamma_4\gamma_\lambda e_\lambda$ goes into the velocity operator $\boldsymbol{\Pi}/mc$ in the nonrelativistic limit. In the presence of an external magnetic field the velocity operator $\boldsymbol{\Pi}$ is given by $\boldsymbol{p} - (e/c)\boldsymbol{A}_{\text{ext}}$, where $\boldsymbol{A}_{\text{ext}} = \frac{1}{2}\boldsymbol{H}\times\boldsymbol{r}$.

As usual, the wave functions for the neutrino-antineutrino pair are taken to be plane waves; for instance, the neutrino emitted at the space time point y in Figure 1*a* is represented by

$$\psi_\nu(y) = \Omega_\nu^{-1/2} e^{iqy} U_\nu(q)\,, \tag{13}$$

where $U_\nu(q)$ is the neutrino spinor with four-momentum q. By considerations similar to those described above for the velocity operator $\boldsymbol{\Pi}$ it can easily be shown that the nonrelativistic limit of the product $\gamma_4\Gamma_\mu(\bar{U}_\nu\Gamma_\mu U_\nu)$ for the direct diagram yields

$$\begin{gathered}(\psi_f^*\gamma_4\Gamma_\mu \exp[i(\boldsymbol{p}+\boldsymbol{q})\cdot\boldsymbol{y}/\hbar]\psi_I)(\bar{U}_\nu\Gamma_\mu U_\nu) \approx (\psi_f^* L_\alpha \exp[i(\boldsymbol{p}+\boldsymbol{q})\cdot\boldsymbol{y}/\hbar]\psi_I)(\bar{U}_\nu\Gamma_\alpha U_\nu)\,,\\ L_\alpha = (L_k, L_4)\,, \qquad L_k = i\sigma_k\,, \qquad L_4 = 1 \quad (k = 1, 2, 3)\,,\end{gathered} \tag{14}$$

where σ_k are the Pauli spin matrices.

Inserting both equation (12) and equation (14) into equations (8) and (9) and adding the resulting expressions, we obtain for the total S-matrix element

$$\begin{aligned} S_{fi} &= S_0\Lambda_\alpha[\bar{U}_\nu(p)\Gamma_\alpha U_\nu(q)]\delta(E_i + E_\gamma - E_f - E_\nu - E_{\bar\nu})\,,\\ \Lambda_\alpha &= \sum_I \left\{\frac{\langle f|L_\alpha \exp[i(\boldsymbol{p}+\boldsymbol{q})\cdot\boldsymbol{y}/\hbar]|I\rangle\langle I|(mc)^{-1}\boldsymbol{\Pi}\cdot\boldsymbol{e}\exp(i\boldsymbol{k}_\gamma\cdot\boldsymbol{x})|i\rangle}{E_i - E_f + E_\gamma + i\eta}\right.\\ &\qquad \left. + \frac{\langle f|(mc)^{-1}\boldsymbol{\Pi}\cdot\boldsymbol{e}\exp(i\boldsymbol{k}_\gamma\cdot\boldsymbol{x})|I\rangle\langle I|L_\alpha\exp[i(\boldsymbol{p}+\boldsymbol{q})\cdot\boldsymbol{y}/\hbar]|i\rangle}{E_I - E_f + E_\gamma + i\eta}\right\},\\ S_0 &\equiv \frac{2\pi e g}{2^{1/2}\Omega_\nu} N_\gamma^{1/2}\,, \end{aligned} \tag{15}$$

where N_γ is the normalization factor for the electromagnetic potential A_μ defined in equation (10).

The square of the absolute value of the S-matrix element S_{fi} is given by

$$|S_{fi}|^2 = \frac{S_0^2 T}{2\pi\hbar}\delta(E_i + E_\gamma - E_f - E_\nu - E_{\bar\nu})\Lambda_\alpha\Lambda_\beta^*[\bar{U}_\nu^{(r)}\Gamma_\alpha U_\nu^{(s)}][\bar{U}_\nu^{(r)}\Gamma_\beta U_\nu^{(s)}]^\dagger\,, \tag{16}$$

where we have used the relation

$$\delta^2(E) = (2\pi\hbar)^{-1}T\delta(E)\,.$$

After summing on the neutrino spin indices and making use of the projection operators, we obtain

$$\begin{aligned}|S_{fi}|^2 &= S_0^2\delta(E_i + E_\gamma - E_f - E_\nu - E_{\bar\nu})\Lambda_\alpha\Lambda_\beta^*\frac{c^2}{4E_pE_q}p_\mu q_\lambda\,\mathrm{Tr}\,\{\Gamma_\alpha\gamma_\lambda\tilde{\Gamma}_\beta\gamma_\mu\}\,,\\ \tilde{\Gamma}_\alpha &\equiv \gamma_4\Gamma^\dagger\gamma_4\,.\end{aligned} \tag{17}$$

We note that equation (17) still depends on p and q, the neutrino and antineutrino four-momenta. The sum over the neutrino pair momenta is performed by first multiplying by the identity

$$\int d^3Q\delta^3(p + q - Q) = 1 , \qquad Q_\mu = \left\{p + q, \frac{i}{c}(E_i + E_\gamma - E_f)\right\},$$

then introducing

$$\sum_p \sum_q \rightarrow \frac{\Omega_\gamma^{\,2}}{(2\pi\hbar)^6} \int d^3p \int d^3q ,$$

and finally using the Lenard relation

$$\int d^3p \int d^3q \frac{cp_\mu}{2E_p} \frac{cp_\lambda}{2E_q} \delta^4(p + q - Q) = \frac{\pi}{24} [Q_\mu{}^2\delta_{\mu\lambda} + 2Q_\lambda Q_\mu]\theta(Q_0)\theta(-Q_\mu{}^2) . \tag{18}$$

The final result is

$$P \equiv \frac{|S_{fi}|^2}{\Omega T} = \frac{4}{3}\frac{\pi^2e^2g^2}{\hbar c}\frac{N_\gamma}{\Omega}\frac{1}{(2\pi\hbar)^6}\int d^3Q\Lambda_\alpha\Lambda_\beta{}^*(-1)^{1+\delta_{\beta 4}}\{Q_\mu{}^2\delta_{\alpha\beta} - Q_\alpha Q_\beta\}\theta(Q_0)\theta(-Q_\mu{}^2) \tag{19}$$

where P is the probability for transition per unit time per unit volume.

Summing over the final electron spin states and performing an average over the initial electron spin states, we obtain

$$L = \tilde{L}\left[\frac{L_x}{2\pi\hbar}\right]^2\left[\frac{L_z}{2\pi\hbar}\right]^2 \sum_{n=0}^{\infty}\sum_{s=1}^{2}\sum_{n'=0}^{\infty}\sum_{s'=1}^{2}\int_{-\infty}^{\infty} dp_z \int_0^{\xi} dp_x \int_{-\infty}^{\infty} dp_z' \int_0^{\xi} dp_x' f_{ns}[1 - f_{n's'}]$$

$$\times \int d^3k_\gamma N_\gamma(\omega)[E_{ns} + E_\gamma - E_{n's'}]\mathscr{F}(\omega) \int d^3Q\Lambda_\alpha\Lambda_\beta{}^*(-1)^{1+\delta_{\beta 4}}[Q^2\delta_{\alpha\beta} - Q_\alpha Q_\beta]\theta(Q_0)\theta(-Q^2) ,$$

$$\tilde{L} = \tfrac{1}{2}\pi^2 g^2 \tfrac{4}{3}\pi\alpha \frac{N_\gamma}{\Omega}\frac{1}{(2\pi\hbar)^6}\frac{\Omega_\gamma}{(2\pi)^3} , \qquad \xi \equiv \frac{eH}{c} Ly . \tag{20}$$

We now calculate the matrix elements contained in the sum Λ_α over the intermediate states. Consider first the matrix element for the neutrino pair emission at the spacetime point y in Figure 1a:

$$M_\alpha{}^\nu = \langle f|\exp(iQ\cdot y/\hbar)L_\alpha|I\rangle , \tag{21}$$

where Q denotes the neutrino-pair momenta ($p + q$). The subscript α is the Lorentz index referred to the electrons, and the superscript ν represents the neutrinos. The electron wave function takes the form

$$\begin{aligned} \psi(x) &= (L_xL_z)^{-1/2}\exp[i(k_1x_1 + k_3x_3)]\exp(-\xi^2/2)U_{ns}(\xi) , \\ \xi &= x_2\zeta^{1/2} + k_1\zeta^{-1/2} , \qquad \zeta \equiv \Theta\,\lambda^{-2} , \qquad \lambda = \hbar/mc , \\ p_i &= \hbar k_i \quad (i = 1, 2, 3) . \end{aligned} \tag{22}$$

Inserting equation (22) into equation (21) and performing the necessary integrations, we obtain

$$M_\alpha{}^\nu = \left[\frac{2\pi\hbar}{(L_xL_z)^{1/2}}\right]^2\delta(p_1'' - p_1' + Q_1)\delta(p_3'' - p_3' + Q_3)\langle s'|L_\alpha|s''\rangle\langle n'|\exp(iQ_2Y_2)|n''\rangle ,$$

$$\langle n'|\exp(iQ_2y_2)|n''\rangle \equiv \int_{-\infty}^{\infty} dy_2 \exp(iQ_2y_2)\exp[-(\xi'^2 + \xi''^2)/2]U_{n'}{}^*(\xi')U_{n''}(\xi'') ,$$

$$\xi' = y_2\zeta^{1/2} + p_1'\zeta^{-1/2} = \xi'' + Q_1\zeta^{-1/2} , \qquad \xi'' = y_2\zeta^{1/2} + p_1''\zeta^{-1/2} . \tag{23}$$

Let us now compute the matrix element for the absorption of the transverse plasmon at the vertix x, namely

$$M^\gamma = \langle I|(mc)^{-1}\mathbf{\Pi}\cdot\boldsymbol{e}\exp(ik_\gamma\cdot x)|i\rangle , \tag{24}$$

where $\mathbf{\Pi}$ is the velocity operator for the electrons, $\boldsymbol{e}$ is the polarization vector for the transverse plasmons, and the index γ represents all quantities referred to the plasmons.

Substituting equation (22) into equation (24) and integrating over the spatial coordinates, we obtain

$$M^{\gamma} = \left[\frac{2\pi\hbar}{(L_x L_z)^{1/2}}\right]^2 \delta(p_1 - p_1'' + \hbar k_\gamma \sin\theta)\delta(p_3 - p_3'' + \hbar k_\gamma \cos\theta)\delta_{s''s} \times \{(\tfrac{1}{2}\Theta)^{1/2}[(n+1)^{1/2}e_-\delta_{n'',n+1} + n^{1/2}e_+\delta_{n'',n-1}] + \langle n''|x|n\rangle e_3\}, \tag{25}$$

where $x = p_3/mc$ is the dimensionless momentum component for the electrons along the magnetic field.

To evaluate the contribution to Λ_α we first replace the sum over the intermediate states I by

$$\sum_I \to \frac{L_z}{2\pi\hbar}\int_{-\infty}^{\infty} dp_3'' \frac{L_x}{2\pi\hbar}\int dp_1'' \sum_{n''=0}^{\infty}\sum_{s''}$$

[where the upper limit of the second integral is $(eH/c)L_y$], and then use the momentum delta functions to integrate over the intermediate continuum states along the magnetic field, and finally perform a discrete sum over the harmonic oscillator states. After summing on the intermediate spin states, the final result for Λ_α arising from the contribution due to the direct diagram can be written as

$$\Lambda_\alpha^D = \frac{(2\pi\hbar)^2}{L_x L_z}\delta(p_1 - p_1' + Q_1 + \hbar k_\gamma \sin\theta)\delta(p_3 - p_3' + Q_3 + \hbar k_\gamma\cos\theta)\delta_{s''s} \times \left\{(\tfrac{1}{2}\Theta)^{1/2}\left[\frac{\langle n'|\exp(iQ_2y_2)|n+1\rangle(n+1)^{1/2}e_-}{E_{n+1}^{s''}(p_3'-Q_3) - E_n^s(p_3) - E_\gamma + i\eta} + \frac{\langle n'|\exp(iQ_2y_2)|n-1\rangle n^{1/2}e_+}{E_{n-1}^{s''}(p_3'-Q_3) - E_n^s(p_3) - E_\gamma + i\eta}\right] + \frac{\langle n'|\exp(iQ_2y_2)|n\rangle x e_3}{E_n^{s''}(p_3'-Q_3) - E_n^s(p_3) - E_\gamma + i\eta}\right\}\langle s'|L_\alpha|s''\rangle, \tag{26}$$

where $E_\gamma = \hbar\omega/mc^2$ is the plasmon energy, and the energy of the electrons is given by

$$E_n^s(p_3) = (n + \tfrac{1}{2} + \tfrac{1}{2}s)mc^2\Theta + p_3^2/2m = mc^2\epsilon(n, s, x),$$
$$\epsilon(n, s, x) = (n + \tfrac{1}{2} + \tfrac{1}{2}s)\Theta + \tfrac{1}{2}x^2, \qquad x = p_3/mc.$$

The contribution to Λ_α arising from the exchange diagram is calculated in exactly the same fashion, and the total matrix element Λ_α for the process is then

$$\Lambda_\alpha = \frac{(2\pi\hbar)^2}{L_x L_z}\delta(p_1 - p_1' + Q_1 + \hbar k_\gamma\sin\theta)\delta(p_3 - p_3' + Q_3 + \hbar k_\gamma\cos\theta)M_\alpha,$$

$$M_\alpha = L_\alpha(s, s')\mathcal{M}(n, p_3; n', p_3'; Q_3, \theta, \Theta, \omega),$$

$$\mathcal{M} \equiv \mathcal{M}_D + \mathcal{M}_E,$$

$$\mathcal{M}_D = (\tfrac{1}{2}\Theta)^{1/2}\left\{\frac{\langle n'|R|n-1\rangle n^{1/2}e_+}{E_{n-1}(p_3'-Q_3) - E_n(p_3) - E_\gamma + i\eta} + \frac{\langle n'|R|n+1\rangle(n+1)^{1/2}e_-}{E_{n+1}(p_3'-Q_3) - E_n(p_3) - E_\gamma + i\eta} + \frac{\langle n'|R|n\rangle x e_3}{E_{n+1}(p_3'-Q_3) - E_n(p_3) - E_\gamma + i\eta}\right\},$$

$$\mathcal{M}_E = (\tfrac{1}{2}\Theta)^{1/2}\left\{\frac{\langle n'+1|R|n\rangle(n'+1)^{1/2}e_+}{E_{n'+1}(p_3+Q_3) - E_{n'}(p_3') + E_\gamma + i\eta} + \frac{\langle n'-1|R|n\rangle n'^{1/2}e_-}{E_{n'-1}(p_3+Q_3) - E_{n'}(p_3') + E_\gamma + i\eta} + \frac{\langle n'|R|n\rangle x' e_3}{E_{n'}(p_3+Q_3) - E_{n'}(p_3') + E_\gamma + i\eta}\right\},$$

$$X = p_3/mc, \qquad x' = p_3'/mc, \qquad E_\gamma = \hbar\omega/mc^2,$$

$$L_\alpha(s, s') \equiv \langle s'|L_\alpha|s\rangle, \qquad R \equiv \exp(iQ_2y_2). \tag{27}$$

The spin dependence of the matrix element Λ_α is represented by the factor $L_\alpha(s, s')$ whereas the matrix element $\mathcal{M}$ is now independent of the electron spins.

The integration over the plasmon momenta d^3k_γ is performed by multiplying equation (20) by the identity

$$\int d\omega\,\delta[\omega - \omega_l(k_\gamma)] = 1$$

to take into account the dispersion of ω through the relation

$$c^2k_\gamma^2 = \omega^2 n_l^2 ,$$

where $n_l(k_\gamma, \omega)$ is the plasmon index of refraction for the mode of excitation l. The particular form of $n_l(k_\gamma, \omega)$ depends on the plasma model, which we shall not specify at this stage in order to make our formulation for the energy loss rate as general as possible within the framework of the nonrelativistic approximation. Inserting the general expression for N_γ from equation (10) into equation (20) and making use of the relation

$$\frac{\delta[\omega - \omega_l(k_\gamma)]}{|\partial\|\Lambda_{ij}\|/\partial\omega|} = \frac{\delta[n^2 - n_l^2]}{|\partial\|\Lambda_{ij}\|/\partial n^2|_{\|\Lambda_{ij}\|=0}} ,$$

we obtain

$$\begin{aligned} L = \tilde{L}\frac{L_xL_z}{(2\pi\hbar)^2} \sum_{n=0}^{\infty}\sum_{n'=0}^{\infty}\sum_{s=1}^{2}\sum_{s'=1}^{2}\int_{-\infty}^{\infty} dp_3 \int_{-\infty}^{\infty} dp_3' \int_0^{\xi} dp_1 \int_0^{\xi} dp_1' f_{ns}(p_3) \\ \times [1 - f_{n's'}(p_3')] \int d\Omega_{k_\gamma} \int \omega d\omega n_l(\omega)\mathscr{F}(\omega) \left| \frac{\mathrm{Tr}\,\lambda_{ij}}{\partial\|\Lambda_{ij}\|/\partial n^2} \right|_{\|\Lambda_{ij}\|=0} \\ \times [E_{ns}(p_3) + \hbar\omega - E_{n's'}(p_3')] \int d^3Q(-1)^{1+\delta_{\beta 4}}[Q^2\delta_{\alpha\beta} - Q_\alpha Q_\beta]L_\alpha(s, s')L_\beta^*(s, s') \\ \times \mathscr{M}\mathscr{M}^*\delta\left[p_1 - p_1' + Q_1 + \frac{\hbar}{c}\omega n_l(\omega)\sin\theta\right]\delta\left[p_3 - p_3' + Q_3 + \frac{\hbar}{c}\omega n_l(\omega)\cos\theta\right]\theta(Q_0)\theta(-Q^2) . \end{aligned} \tag{28}$$

We can now integrate over all the particle momenta. Successive integrations over the electron momenta dp_1' and dp_1 immediately yield unity and $(eH/c)L_\gamma$, respectively. We then use the delta function $\delta[P_3 - p_3' + Q_3 + (\hbar\omega/c)n_l(\omega)\cos\theta]$ to integrate over the neutrino-pair momenta dQ_3 along the magnetic field.

Finally we integrate over the neutrino-pair momenta dQ_1dQ_2 perpendicular to the magnetic field by using the two step functions $\theta(Q_0)\theta(-Q^2)$. The integrations are elementary and can be performed most easily by introducing plane polar coordinates. After summing on the initial and the final electron spin states over the Fermi and Gamow-Teller transition, the final result for the differential neutrino luminosity per unit solid angle may be cast in the form

$$\begin{aligned} L(\theta) = L_0\Theta \sum_{n=0}^{\infty}\sum_{n'=0}^{\infty}\int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dx' \int_0^{\infty} \omega d\omega n_l(\omega)\mathscr{F}(\omega)\left|\frac{\mathrm{Tr}\,\lambda_{ij}}{\partial\|\Lambda_{ij}\|/\partial n^2}\right|_{\|\Lambda_{ij}\|=0} \\ \times \int_0^1 d\rho[A(n, n') - \rho B(n, n')]|\mathscr{M}_l|^2 , \end{aligned} \tag{29a}$$

$$\begin{aligned} A(n, n') &= Q^2(n, n')\epsilon(n, n')[\epsilon^2(n, n') + Q_3^2]\{f_n(x)[1 - f_{n'}(x')] + f_{n+1}(x)[1 - f_{n'+1}(x')]\} \\ &\quad + 2\{Q^4(n+1, n')\epsilon(n+1, n')f_{n+1}(x)[1 - f_{n'}(x')] + Q^4(n, n'+1)\epsilon(n, n'+1)f_n(x)[1 - f_{n'+\alpha}(x')]\} , \\ B(n, n') &= Q^4(n+1, n')\epsilon(n+1, n')f_{n+1}(x)[1 - f_{n'}(x')] + Q^4(n, n'+1)\epsilon(n, n'+1)f_n(x)[1 - f_{n'+1}(x')] , \\ \epsilon(n, n') &= \tfrac{1}{2}(x^2 - x'^2) + (n - n')\Theta + \omega , \\ Q^2(n, n') &= \epsilon^2(n, n') - Q_3^2 , \qquad Q_3 = x - x' + \omega n_l(\omega)\cos\theta , \\ f_n(x) &= \left\{1 + \exp\left[\frac{\epsilon_n(x) - \bar{\mu}}{\varphi}\right]\right\}^{-1} , \qquad \varphi = kT/mc^2 , \\ \epsilon_n(x) &= \frac{x^2}{2} + n\Theta , \qquad x = p_3/mc , \qquad x' = p_3'/mc , \\ \mathscr{M}_l &= \mathscr{M}_l^D + \mathscr{M}_l^E , \\ \mathscr{M}_l^D &= (\tfrac{1}{2}\Theta)^{1/2}\left\{\frac{(n+1)^{1/2}\langle n'|R|n+1\rangle[e_1^l - ie_2^l]}{\Theta - \omega + x\omega\mu n_l(\omega) + \frac{1}{2}\omega^2\mu^2 n_l^2(\omega) + i\eta} + \frac{n^{1/2}\langle n'|R|n-1\rangle[e_1^l + ie_2^l]}{-\Theta - \omega + x\omega\mu n_l(\omega) + \frac{1}{2}\omega^2\mu^2 n_l^2(\omega) + i\eta}\right\} \\ &\quad + \frac{\langle n'|R|n\rangle x e_3^l}{\omega + x\omega\mu n_l(\omega) + \frac{1}{2}\omega^2\mu^2 n_l^2(\omega) + i\eta} , \end{aligned}$$

$$\mathscr{M}_l^E = (\tfrac{1}{2}\Theta)^{1/2}\left\{\frac{(n'+1)^{1/2}\langle n'+1|R|n\rangle[e_1{}^l + ie_2{}^l]}{\Theta + \omega - x'\omega\mu n_l(\omega) + \frac{1}{2}\omega^2\mu^2 n_l{}^2(\omega) + i\eta} + \frac{n'^{1/2}\langle n'-1|R|n\rangle[e_1{}^l - ie_2{}^l]}{\omega - \Theta - x'\omega\mu n_l(\omega) + \frac{1}{2}\omega^2\mu^2 n_l{}^2(\omega) + i\eta}\right\}$$

$$+ \frac{\langle n'|R|n\rangle x' e_3{}^l}{\omega - x'\omega\mu n_l(\omega) + \frac{1}{2}\omega^2\mu^2 n_l{}^2(\omega) + i\eta},$$

$$R = \exp[iQ_2 y_2], \qquad \mu = \cos\theta, \tag{29b}$$

where the index l represents the plasmon mode of excitation.

III. THE COLLISIONLESS COLD PLASMA MODEL

The excitation frequency $\omega_l(k_y)$ for a strongly magnetized electron plasma has been investigated by several authors (Klimontovich and Silin 1961; Canuto and Chou 1973) and has been discussed by Canuto, Chiuderi, and Chou (1970) in connection with the plasmon neutrino process. To apply our general result for the photoneutrino luminosity to transverse plasmons, we make few approximations to obtain explicit functional forms for the excitation frequencies. We neglect both spatial dispersion due to electron pressure for an electron gas of arbitrary degeneracy and the effects of spin and collisions on the dielectric tensor. The effect of spatial dispersion has been shown by Adams, Ruderman, and Wu (1963) to be unimportant in the high-density regime such as the interiors of neutron stars and dwarf stars whereas the spin effects on the dielectric tensor are negligible (Burt and Wahlquist 1962). In a superstrong magnetoactive electron plasma, the cyclotron frequency is so high that particle correlations are due to magnetic fields rather than collisions; hence the effect of collisions on the dielectric tensor can also be neglected. Furthermore, we consider only purely transverse and purely longitudinal modes. In this paper we shall concentrate on the transverse modes whose effects on the photoneutrino luminosity are more significant.

In view of the approximations delineated above we shall therefore proceed to compute the neutrino-pair energy loss rate with the cold, collisionless plasma model. In the notation of Stix (1962), the dielectric tensor can be written

$$\epsilon_{ij} = \begin{bmatrix} S & -iD & 0 \\ iD & S & 0 \\ 0 & 0 & P \end{bmatrix}, \tag{30}$$

$$S = \tfrac{1}{2}(R+L), \qquad D = \tfrac{1}{2}(R-L), \qquad P = 1 - \frac{\omega_p{}^2}{\omega^2}, \qquad \omega_H = \frac{eH}{mc},$$

$$R = 1 - \frac{\omega_p{}^2}{\omega(\omega-\omega_H)}, \qquad L = 1 - \frac{\omega_p{}^2}{\omega(\omega+\omega_H)}, \qquad \omega_p{}^2 = \frac{4\pi N_e{}^2}{m},$$

where N is the electron number density and ω_p is the plasma frequency. The index of refraction satisfies the biquadratic equation

$$An^4 - Bn^2 + C = 0,$$

$$A = S\sin^2\theta + P\cos^2\theta, \qquad B = RL\sin^2\theta + \mathrm{SP}(1+\cos^2\theta), \qquad C = PRL. \tag{31}$$

The solutions are given by

$$n^2 = \frac{B \pm F}{2A}, \qquad F^2 = B^2 - 4AC.$$

Equation (31) can be cast in a more transparent form (Astron 1950; Allis, Buchsbaum, and Bers 1963)

$$\tan^2\theta = -\frac{P(n^2-R)(n^2-L)}{(Sn^2-RL)(n^2-P)}, \tag{32}$$

which shows that for propagation along the magnetic field ($\theta = 0$) we have either $n_O{}^2 = R$ for the ordinary mode O or $n_x{}^2 = L$ for the extraordinary mode X. Similarly for propagation across the magnetic field ($\theta = \pi/2$) we have the ordinary mode $n_O{}^2 = P$ and the extraordinary mode $n_X{}^2 = RL/S$. The latter, being a linearly polarized mixed mode, is partially transverse and partially longitudinal. It can be easily shown that $\mathrm{Tr}(\lambda_{ij})$ is given by

$$\mathrm{Tr}(\lambda_{ij}) = n^4 - (P + A + 2S)n^2 + RL + 2PS.$$

The general form of the polarization vector $\boldsymbol{e}$ has been considered by Melrose (1968), whose result is

$$e_i = \frac{\lambda_{ij}c_j}{[c_i{}^*\lambda_{ij}c_j\,\mathrm{Tr}(\lambda_{ij})]^{1/2}}.$$

The index l specifying the various modes of excitation is implicitly contained in λ_{ij} through the index of refraction n_l. The constant parameters c_j are a set of complex numbers with the constraint that one of them should not vanish. Let us choose

$$c = (0, i, 0) .$$

We then obtain after some straightforward computation

$$e_l = (1 + G_l^2)^{-1/2}\left\{\frac{D(P - n_l^2 \sin^2\theta)}{SP - An_l^2},\ i,\ \frac{-Dn_l^2 \sin\theta\cos\theta}{SP - An_l^2}\right\}, \qquad G_l = \frac{PD\cos\theta}{SP - An_l^2} . \tag{33}$$

It follows that

$$\left|\frac{\mathrm{Tr}(\lambda_{ij})}{\partial\|\Lambda_{ij}\|/\partial n^2}\right|_{\|\Lambda_{ij}\|=0} = \left|\frac{n^4 - (P + A + 2S)n^2 + (RL + 2SP)}{\pm F}\right| , \tag{34}$$

since

$$2An^2 - B = \pm F$$

Inserting equations (33) and (34) into equation (29a), we then obtain the differential luminosity per unit solid angle for incoming plasmons making an angle θ relative to the external magnetic field. The resulting expressions are quite involved because of the complicated angular dependence of the polarization vector and the normalization factor N_γ. To bring out the physical significance more clearly, it is fruitful to study the important special cases of parallel and perpendicular propagation. In what follows we shall consider these two cases in detail.

a) Propagation Parallel to the Magnetic Field

For propagation parellel to the magnetic field ($\theta = 0$) the polarization vector becomes

$$e_O = 2^{-1/2}(1, i, 0) , \qquad e_X = 2^{-1/2}(1, -i, 0) , \tag{35}$$

where the subscripts O and X denote the ordinary and the extraordinary modes, respectively. These are the well known circularly polarized transverse waves. The ordinary mode is right-hand circularly polarized whereas the extraordinary mode is left-hand circularly polarized. It can be shown for both of these waves the normalization factor

$$\left|\frac{\mathrm{Tr}(\lambda_{ij})}{\partial\|\Lambda_{ij}\|/\partial n^2}\right|_{\|\Lambda_{ij}\|=0}$$

simply reduces to unity. The maxtrix element for the ordinary mode is then

$$\begin{aligned}
\mathscr{M}_O &= \Theta^{1/2}\left\{\frac{(n+1)^{1/2}\langle n'|R|n+1\rangle}{\omega_H - \omega + x\omega n_O + \frac{1}{2}\omega^2 n_O^2 + i\eta} + \frac{n'^{1/2}\langle n'-1|R|n\rangle}{\omega - \omega_H - x'\omega n_O + \frac{1}{2}\omega^2 n_O^2 + i\eta}\right\} , \\
Q_3 &= x - x' + \omega n_O(\omega) , \qquad n_O^2(\omega) = 1 - \frac{\omega_p^2}{\omega(\omega - \omega_H)} , \\
e_+^O &= 0 , \qquad e_-^O = \sqrt{2} ,
\end{aligned} \tag{36}$$

since

$$e_\pm^l = e_1^l \pm ie_2^l$$

On the other hand, the matrix element for the extraordinary mode becomes

$$\begin{aligned}
\mathscr{M}_X &= \Theta^{1/2}\left\{\frac{(n'+1)^{1/2}\langle n'+1|R|n\rangle}{\omega + \omega_H - x'\omega n_X + \frac{1}{2}\omega^2 n_X^2 + i\eta} - \frac{n^{1/2}\langle n'|R|n-1\rangle}{\omega + \omega_H - x\omega n_X - \frac{1}{2}\omega^2 n_X^2 - i\eta}\right\} , \\
Q_3 &= x - x' + \omega n_X(\omega) , \qquad n_X^2(\omega) = 1 - \frac{\omega_p^2}{\omega(\omega + \omega_H)} , \\
e_+^X &= \sqrt{2} , \qquad e_-^X = 0 .
\end{aligned} \tag{37}$$

b) Propagation Perpendicular to the Magnetic Field

For propagation across the magnetic field, the propagation vector becomes

$$e_O = (0, 0, i) , \qquad e_X = (0, 1, 0) . \tag{38}$$

The normalization factor for both of these modes again reduces to unity. The matrix element for the ordinary mode is given by

$$\mathscr{M}_X\left(\frac{\pi}{2}\right) = i\langle n'|R|n\rangle\left\{\frac{x'}{\omega + i\eta} + \frac{x}{\omega - i\eta}\right\}, \qquad Q_3 = x - x', \qquad e^O{}_{\pm} = 0, \qquad e_3{}^O = i. \tag{39}$$

Similarly for the extraordinary mode we have

$$\mathscr{M}_X\left(\frac{\pi}{2}\right) = i(\tfrac{1}{2}\Theta)^{1/2}\left\{\frac{(n'+1)^{1/2}\langle n'+1|R|n\rangle}{\omega + \omega_H + i\eta} - \frac{n'^{1/2}\langle n'-1|R|n\rangle}{\omega - \omega_H + i\eta} - \frac{n^{1/2}\langle n'|R|n-1\rangle}{\omega + \omega_H - i\eta} + \frac{(n+1)^{1/2}\langle n'|R|n+1\rangle}{\omega - \omega_H - i\eta}\right\},$$

$$Q_3 = x - x', \tag{40}$$

where we have made use of the relations

$$e_{\pm}{}^X = \pm i, \qquad e_3{}^X = 0.$$

We note that although both $\mathscr{M}_O$ and $\mathscr{M}_X$ do not depend on the index of refraction, the energy loss rates are still affected by the presence of the plasma through the factor $\omega n_l(\omega)\mathscr{F}(\omega)$ in the thermal average.

Due to the complexities of the resulting expression for the neutrino luminosity even for these special cases delineated above, it seems that analytic solutions are exceedingly difficult if not entirely impossible. In the next section of this paper we shall therefore present our solutions in numerical form and elaborate in detail their possible applications to neutron star matter and stellar interiors.

IV. RESULTS AND APPLICATIONS

In a magnetoactive electron plasma the neutrino luminosity is modified by both the plasma and the magnetic field. In the presence of plasma excitations the neutrino-pair energy loss rate is reduced whereas the presence of a strong magnetic field imposes the restriction that the interaction of the plasmons with the electrons can lead predominately only to a recoil of the electrons along the magnetic field.

More specifically, for propagation along the magnetic field, the electric vector of the circularly polarized transverse plasma waves rotates in the plane perpendicular to the magnetic field, and it therefore will act on the velocity components of the Landau orbital electrons which, being quantized, present great resistance to any external force.

On the other hand, for propagation across the magnetic field, the electric vector of the linearly polarized transverse waves can be oriented either in the direction of the field (for the ordinary mode) or perpendicular to the field (for the extraordinary mode). Thus, the extraordinary mode is similarly affected by the magnetic field whereas the ordinary mode is unchanged.

Now the energy carried away by the neutrino pair is supplied by the electrons. In the presence of plasma excitations the energy available for the neutrino-pair is diminished and the resulting neutrino rate is greatly reduced at high densities since the plasmon quantum is proportional to the square root of the density. It is then expected that in the presence of a magnetized electron plasma the photoneutrino luminosity is significantly reduced.

The reduction of the neutrino luminosity at low densities, where the plasma frequency is small, is due mainly to the magnetic field whereas at high densities the reduction is dominated by the plasmons. Morever, the reduction of the photoneutrino rate is clearly anisotropic and the effect is more pronounced for the circularly polarized transverse plasma waves traveling along the external magnetic field. In what follows we shall therefore focus our attention on these important special modes of excitation.

To appreciate the interesting features of these waves in greater detail, we shall present some numerical results for the extraordinary mode for propagation along the magnetic field.

The index of refraction for this mode is given by the third of equations (37). The numerical integration is performed by imposing the condition that the index of refraction must be real. This requirement sets a lower limit and the pole at $\omega = 0$ is automatically avoided. We assume that the electron gas is completely degenerate and hereby replace the Fermi functions by step functions. It seems that this procedure is reasonable in view of the fact that we have used a collisionless cold plasma model. The temperature dependence is then essentially dictated by the Bose distribution of the plasmons. For application to neutron star matter, the approximations introduced above are somewhat justified.

Examples of the results can be found in Figures 2, 3, and 4 and Tables 1 and 2 which give the differential luminosity $L(\theta = 0)$ per unit solid angle per unit volume as a function of density ρ_6/μ_e at temperatures $T_8 = 2.5$, 3.85, and 10, respectively. For purpose of comparison we have also included the results of Beaudet, Petrosian, and Salpeter (1967) for an isotropic plasma without magnetic field together with the results of Canuto *et al.* (1974) who considered the effect of the magnetic field but neglected the presence of the plasma.

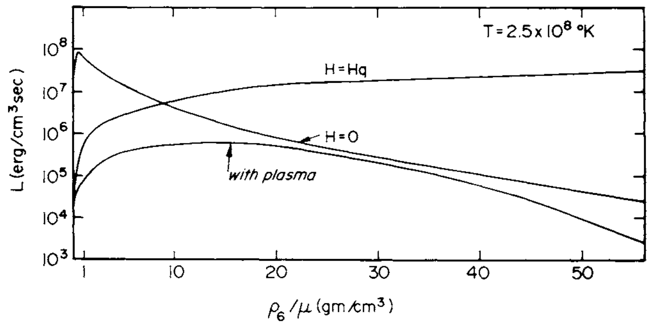

FIG. 2.—Neutrino luminosity (ergs cm^{-3} s^{-1}) as a function of density ρ_6/μ_e (g cm^{-3}) for plasma waves propagating along the external magnetic field (in the extraordinary mode X) at temperature $T = 2.5 \times 10^8$ K and $H = H_q = 4.41 \times 10^{13}$ gauss. The curve marked $H = H_q$ is due to Canuto *et al.* (1974), who neglected plasma effects. The curve marked $H = 0$ is due to Beaudet *et al.* (1967) who included the effects of an isotropic plasma without magnetic field.

TABLE 1

NEUTRINO LUMINOSITY AS A FUNCTION OF DENSITY ρ_6/μ_e
($T_8 = 3.85$, $H = Hq$)

ρ_6/μ_e	$L_p(H)$*	$L(H)$†	$L_p(0)$‡
1.44........	8.7×10^6	2.55×10^7	4.81×10^9
2.76........	9.6×10^6	4.08×10^7	4.09×10^9
4.66........	1.17×10^7	8.6×10^7	3.08×10^9
12.9.........	4.13×10^7	3.36×10^8	1.15×10^9
23.6.........	6.05×10^7	7.3×10^8	4.7×10^8
36.5.........	4.51×10^7	4.38×10^8	2.09×10^8
46.7.........	3.24×10^7	3.3×10^8	1.22×10^8

* $L_p(H)$: with plasma and magnetic field (present work), for the extraordinary mode X propagating along the magnetic field.

† $L(H)$: with magnetic field, no plasma (Canuto *et al.* 1974).

‡ $L_p(0)$: with plasma, no magnetic field (Beaudet *et al.* 1967).

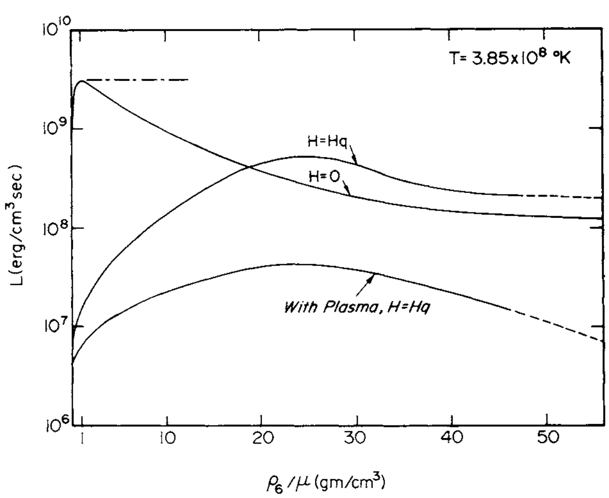

FIG. 3

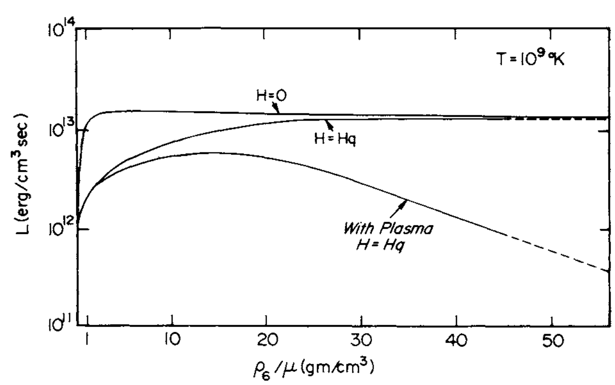

FIG. 4

FIG. 3.—Same as Fig. 2, for $T = 3.85 \times 10^8$ K

FIG. 4.—Same as Fig. 2, for $T = 10^9$ K

TABLE 2

NEUTRINO LUMINOSITY AS A FUNCTION OF DENSITY ρ_6/μ_e
($T_8 = 10$, $H = H_q$)

ρ_6/μ_e	$L_p(H)$*	$L(H)$†	$L_p(0)$‡
1.44........	3.96×10^{12}	4.13×10^{12}	1.31×10^{13}
2.76........	4.13×10^{12}	4.44×10^{12}	1.67×10^{13}
4.66........	4.8×10^{12}	4.93×10^{12}	1.85×10^{13}
12.9.........	7.6×10^{12}	9.5×10^{12}	1.77×10^{13}
23.7.........	6.82×10^{12}	1.09×10^{13}	1.5×10^{13}
36.5.........	2.16×10^{12}	1.09×10^{13}	1.24×10^{13}
46.7.........	1.46×10^{12}	1.1×10^{13}	1.08×10^{13}

* $L_p(H)$: with plasma and magnetic field (present work), for the extraordinary mode X propagating along the magnetic field.

† $L(H)$: with magnetic field, no plasma (Canuto *et al.* 1974).

‡ $L_p(0)$: with plasma, no magnetic field (Beaudet *et al.* 1967).

At relatively low densities our results are almost the same as those of Canuto *et al.* (1974). This is to be expected because the reduction of the neutrino rate at low densities ($\rho_6/\mu_e \lesssim 20$) is due to the magnetic field rather than to plasma effects. However, at high densities ($\rho_6/\mu_e \gtrsim 20$) the neutrino rate is significantly reduced because a greater part of the energy of the electrons (resulting from a Landau transition) is converted into plasmon excitations instead of being carried off by the neutrinos. Thus the presence of a magnetized electron plasma tends to reduce the photoneutrino luminosity, and the effects are more pronounced at both the high-density end (due to plasmon excitation) and at the low-density end (due to the magnetic field).

In the presence of a strong magnetic field the most important other neutrino reactions which depend on the direct coupling of the leptonic weak currents $l_\gamma l_\gamma^\dagger$ are the neutrino-pair annihilation, plasmon neutrinos, and the synchrotron neutrino processes. The effects of a strong magnetic field on all these neutrino reactions have been

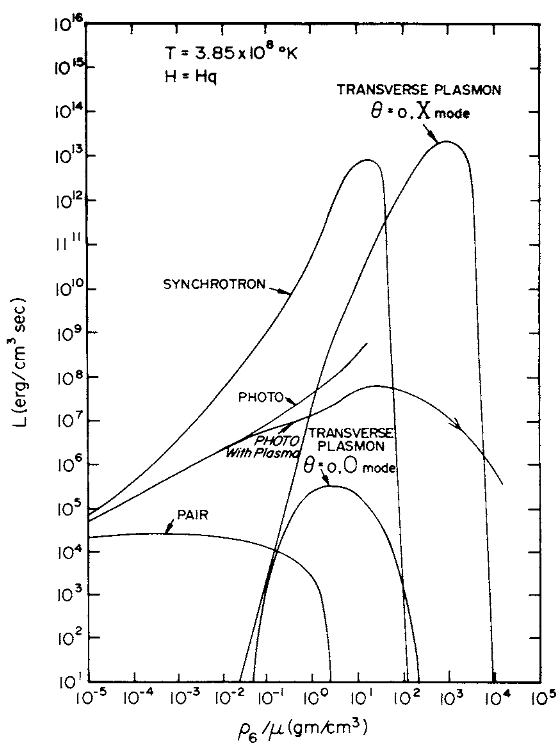

FIG. 5

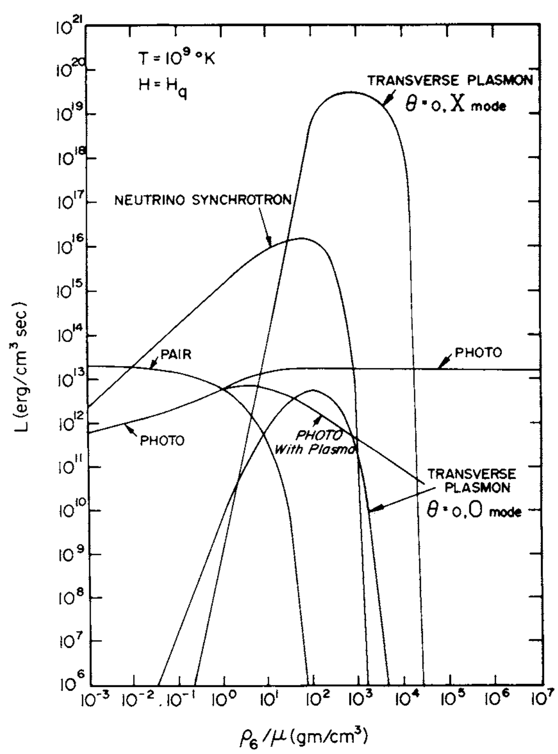

FIG. 6

FIG. 5.—Neutrino luminosity L (ergs cm^{-3} s^{-1}) for: pair, photoneutrino, plasma neutrino, and synchrotron process as a function of density ρ_6/μ_e (g cm^{-3}) for $T = 3.85 \times 10^8$ K and $H = H_q = 4.41 \times 10^{13}$ gauss. The curve marked photo is due to Canuto *et al.* (1974) who neglected the plasma effects. In a magnetoactive electron plasma the photoneutrino luminosity is greatly reduced at high densities because of plasmon excitations, as indicated by the curve marked photo with plasma.

FIG. 6.—Same as Fig. 5, for $T = 10^9$ K

summarized by Canuto *et al.* (1974) except the present work. Thus, it is of some interest to compare the present result with the other competing reactions just mentioned.

Examples of the results are illustrated in Figs. 5 and 6 which give the neutrino rate (in ergs $cm^{-3}\ s^{-1}$) as a functions of density ρ_6/μ_e (in g cm^{-3}) for all the four main reactions at temperatures $T_8 = 3.85$, 10 and field strength $H = H_q = 4.41 \times 10^{13}$ gauss. In general, the photoneutrino and the pair annihilation processes dominate at low densities ($\rho_6/\mu_e < 1$) whereas the synchrotron process competes with the plasmon process at high densities ($1 < \rho_6/\mu_e < 10^4$). This is because the neutrino luminosities for both of the latter two processes are sharply peaked functions of density, and the peaks occur at roughly the same density regime for both temperatures.

The effects of the magnetic field on the plasmon neutrino rate are negligible at high temperatures and densities ($T\sigma \lesssim 10$, $\rho_6/\mu_e \gtrsim 100$) even at the field strength H_q. However, at relatively low temperatures ($T_8 \lesssim 10$) the effects of the field are large. The peak of the extraordinary mode X is shifted toward higher densities relative to the field-free case whereas the neutrino rate for the ordinary mode is greatly suppressed (Canuto, Chiuderi, and Chou 1970). On the other hand, the presence of the magnetic field tends to enhance the pair annihilation rate in the density regime $\rho_6/\mu_e \lesssim 1$ for temperatures up to $T_8 \lesssim 3.85$ beyond which it is significantly reduced for any density (Canuto and Fassio-Canuto 1973*a*). Thus, the total neutrino luminosity due to the four main reactions is even more sharply peaked than the field-free case because of the competition of the synchrotron process with the plasmon process at high densities and the reduction of the photoneutrino rate at both high and low densities due respectively to the presence of the plasma and the magnetic field.

Finally, we think that the following remarks are in order. Without plasma effects, the photoneutrino process was the only one that survived at high densities, thus giving the erroneous impression that a significant cooling could be achieved even when all the other processes had already disappeared. This is not true any longer in the light of the present results.

The presence of plasma has the clear effect of decreasing the luminosity by several orders of magnitude as the density increases, with the net result that no neutrino cooling is to be expected at densities higher than, say, $\rho_6 \sim 10^2$ (Fig. 5).

Admittedly, we have extrapolated outside the region of strict validity of the present nonrelativistic computation. However, the main result of producing a sizable lowering of the luminosity is a physical effect that must persist even if a relativistic computation will ever be doable. Even though, as we explained earlier, the reduction is not unexpected, its exact magnitude could hardly have been guessed without performing the present computation.

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